Transport V → Metals (6.)

→ Metals (follow, "Physics of Materials" Yves Quere)

- free electrons essential,
- but QM also essential. Drude is dubious or general (atomic, i.e.)

i.e. if $d \sim$ lattice scale, for metal

$$p \sim \hbar/d \Rightarrow \epsilon_{QM} \sim \hbar^2/2md^2$$

(~ repulsion)

∴ need $\gtrsim 10$ eV for $T > \epsilon_{QM}$

so for room temp. conductor QM is essential to model. – actually for basics, can take $T=0$

i.e. QM dominated electron gas = metal

- particular focus of QM treatment is Pauli Exclusion Principle, and its effect on electron response → conductivity.

n.b. Exclusion sets distribution function.

so ⇒ review some basics of:

i) [
- QM for free electrons
- density of states]

ii) - Fermi distribution, energy, etc. ⟺ scale re

iii) - conductivity, and Drude redux.

iv) - strength, etc.

Aside:

Example of Fermion Exclusion Driven Pressure: Chandrasekhar Mass.

Takes $M > M_\odot$ to collapse to white dwarf.

Aside: Chandrasekhar Limit — Simple Derivation
(c.f.: Shapiro, Teukolsky)

→ suppose: $\boxed{N}$ Fermions in star of radius R

$\therefore \quad n_{\text{Fermion}} \sim N/R^3$

$\therefore \quad \text{Vol./Fermion} \sim 1/n$ (Pauli exclusion)

$p \sim \hbar/\Delta x \sim \hbar n^{1/3}$ (Heisenberg Uncertainty)

p → Fermion Momentum

$\boxed{\hbar^{-1} p \sim N^{1/3}/R}$

⇒ Fermion energy (per Fermion): $E_F = pc \sim \hbar c \frac{1}{r}$ (i.e. replaces: Thermal energy)

$\sim \hbar c \frac{N^{1/3}}{R}$

(effective repulsion)

Gravitational Energy (per Fermion): $E_{Grav} \sim -\frac{GMm_b}{R}$

m_b → Baryon mass

$M \sim N \; m_B$

Pressure → electron
Mass → Baryon

$\therefore \quad E = E_F + E_G$

$= \frac{\hbar c N^{1/3}}{R} - \frac{G N m_B^2}{R}$

Note QM effects

[redacted]

note: $E = E_F + E_G$

$$= \frac{\hbar c N^{1/3}}{R} - \frac{G N m_B^2}{R}$$

$E > 0$ $\Rightarrow$ decrease E, E_F by increasing R.

but as $E_F \downarrow$, electrons non-relativistic,

$\therefore E_F \sim 1/R^2 \rightarrow$ eqbrm.

$E < 0$ $\Rightarrow$ decrease E without bound by decreasing R $\Rightarrow$ collapse

binding

$\therefore$ eqbrm: $\hbar c N^{1/3} = G N m_B^2$

$$N_{Max} = \left(\hbar c / G m_B^2\right)^{3/2} \sim 2 \times 10^{57} \quad \text{(proton)}$$

Max # particles.

$\therefore$ $M_{chandra} = N_{max} m_B \sim 1.5 M_\odot$

i.e. $M > M_{chandra} \simeq 1.5 M_\odot$

$\Rightarrow N > N_{crit}$ $\Rightarrow$ collapse

QM and Density of States

Key point: $\epsilon_{QM} \sim 10$ eV. $T_{room} \sim .01$ eV.

∴ → can treat metal as zero temperature gas of electrons. (No thermal spread!)

→ distribution set by Pauli Principle.

→ $\partial f_0/\partial E \sim \delta(E-E_0) \Rightarrow \sigma$.

⇒ Density of States is key concept.

i.e. Particles fill up available states

Consider 1D, 3D: Temp relevant at boundary of dist. (*)   box → ion field.

– 1D Particles in a Box.

~ i.e. linear dynamics: organic semi-conductors

~ $\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right)\psi = E\psi$ is TISE

∴

$$E_n = E_0 n^2$$

$$E_0 = \frac{\hbar^2}{2m}\frac{2\pi^2}{L^2}$$ (no spin)

$$\psi_n = \sqrt{\frac{2}{L}} \sin\frac{2\pi x n}{L}\, e^{-iE_n t/\hbar}$$

To count # states ⇒

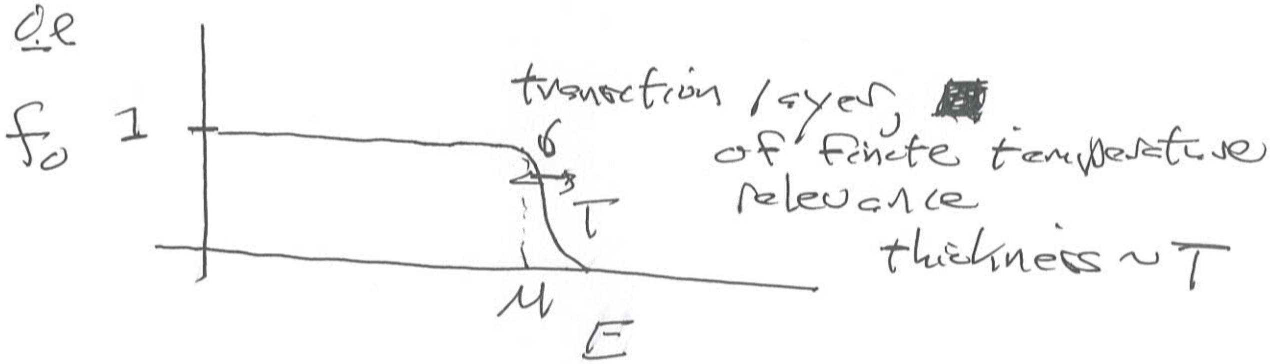
① – count # k's ⇒ integrate in k.

Point is:

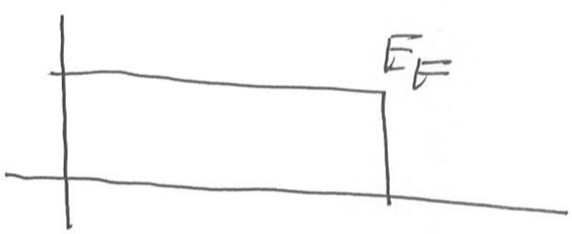
– Fermi-Dirac distribution:

$$f(E) = 1 \Big/ \left[\exp\left(E - \mu / T \right) + 1 \right]$$

i.e



so often, for $\mu \gg T \Rightarrow$



i.e. zero temp. approximation is valid.

Idea, then is that system simply fills up the states $\left(\frac{L}{2\pi}\right)^3$-per-$d^3k$, with 2 occupants per state, due spin $\uparrow\downarrow$.

$\Rightarrow$ $n(E)$ $\Rightarrow$ density of states.

so $\quad E = (\hbar^2/2m)\, k^2$

$$dE = (\hbar^2/\cancel{2}m)\, \cancel{2}k\, dk$$

$\therefore \quad dk = \left(\frac{\hbar^2}{m}\right) / k \quad dE$

$$= \left(\frac{\hbar^2}{m}\right)\left(\frac{2m}{\hbar^2}\right)^{1/2} / \sqrt{E}$$

$$= 2 \left(\frac{2m}{\hbar^2}\right)^{-1/2} / \sqrt{E}$$

so:

$N = \left(\begin{array}{c}\text{states} \\ \text{per } k\end{array}\right) \int dk = \left(\begin{array}{c}\text{states} \\ \text{per } k\end{array}\right) k_F$

\# states
(particles)

$$N = \boxed{2 \cdot \frac{L}{2\pi}} \cdot \int 2 \left(\frac{\hbar^2}{2m}\right)^{1/2} \frac{dE}{\sqrt{E}}$$

$$= 2 \cdot \cancel{2} \cdot \left(\frac{L}{\cancel{2}\pi}\right)\left(\frac{\hbar^2}{2m}\right)^{1/2} \int \frac{dE}{\sqrt{E}}$$

$\therefore \quad N = \int dE \Big/ \sqrt{E_0}\sqrt{E}$

$= \int dE \; n(E)$

$n(E)$ ↓ density of states

$$\boxed{\begin{aligned} n(E) &= 1/\sqrt{E}\sqrt{E_0} \\ E_0 &= (\pi^2/L^2)\,\frac{\hbar^2}{2M} \end{aligned}}$$

$\cong$, with density states normalization

$N(E) = n(E)\, f(E)$

f: FD ← Fermi-Dirac dist.

Now, for $T < \epsilon$, simply fill up states so

$$N = \#\text{particles} = \int_0^{E_F} 1/\sqrt{E\,E_0}$$

(E_F ← Fermi energy)

N(E)

$= 2\left(E_F/E_0\right)^{1/2}$

$$\boxed{E_F = (N/2)^2 E_0 = (N/2)^2 \frac{\pi^2}{L^2}\frac{\hbar^2}{2M}} \sim \frac{N^2}{L^2}$$

$\sim n^2$ (concen.), (N/L).

$\boxed{E_F \sim \frac{\hbar^2}{2M} n^2}$

Now $E_F = \hbar^2/2m \; k_F^2$ (k_F ← Fermi wave vector)

$\boxed{k_F \sim \frac{N}{2} \pi/L}$ → Fermi wave vector ⟺ Momentum.

and $\boxed{v_F \sim \hbar k_F/m}$ → Fermi velocity.

Now, in 3D:

→ have spherical shells in k

∴

→ Fermi sphere!

Fermi-Dirac dist.

i.e. distribution ($N(E)$) now is $\sim n(E)\, f_{FD}(E)$.

so

$$E = (\hbar^2/2m)\, k^2 \qquad k = \frac{2\pi n}{L}$$

$$dE = \left(\frac{\hbar^2}{2m}\right) 2k\, dk$$

and $\underbrace{2}_{\text{spin}} \cdot (2\pi/L)^{-3}$ states / k cell

so

$$N = \int_0^{k_F} 4\pi k^2 dk \left[2 \left(L/2\pi \right)^3 \right]$$

Now: $k^2 = 2mE/\hbar^2$

$$k = (2mE/\hbar^2)^{1/2}$$

$$dk = \left(\frac{2m}{\hbar^3}\right)\frac{1}{2\,(2mE/\hbar^2)^{1/2}}\,dE$$

so//

$$N = 2\,(L/2\pi)^3 \int_0^{E_F} 4\pi\,\frac{2mE}{\hbar^2}\left(\frac{2m}{\hbar^2}\right)\frac{1}{2\left(\frac{2mE}{\hbar^2}\right)^{1/2}}\,dE$$

$$= L^3/2\pi^2 \int_0^{E_F}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}\,dE$$

$$N = \frac{L^3}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^{E_F}\sqrt{E}\,dE$$

$$= \int_0^{E_F} n(E)\,dE$$

$$\boxed{n(E) = \left(L^3/2\pi^2\right)\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}\,dE}$$

density of states

Now, need k_F for momentum

$$N = \int_0^{k_F} 4\pi k^2\,dk\,\left(2\,(L/2\pi)^3\right)$$

$$= 4\pi k^3 \quad 2\,(L/\ )^3$$

$$\frac{N}{L^3} \sim \left(\frac{4\pi}{3}\right) \frac{2}{(2\pi)^3} k_F^3$$

$$\boxed{\left(3\pi^2 N/L^3\right)^{1/3} = k_F}$$ → Fermi wave #

$$\boxed{\begin{aligned} v_F &\sim \frac{\hbar}{m}\left(\frac{3\pi^2 N}{L^3}\right)^{1/3} \\ E_F &\sim \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{L^3}\right)^{2/3} \end{aligned}}$$ → Fermi velocity, energy.

Observe:

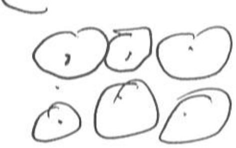
- E_F indep system size, depends concentration only.
  i.e. $\sim (N/L^3)^{2/3} \sim n^{2/3}$.

- this defines a length scale

  i.e. $\frac{4\pi}{3} r_e^3 = L^3/N$

  ↓

  effective volume for single electron

  i.e. 

  so

  $v_F \sim \hbar/m\,(1/r_e)$

so

$v_F \sim 2/r_e^{-1}$ (a.u.)

$E_F \sim 1.8\, r_e^{-2}$ (au) (eV)

so

	r_e	v_F	E_F
Na	3.2	$.5 \times 10^6$ m/sec.	3.2 eV
Al	2	2×10^6 m/sec	11.7 eV

(clearly well above room temp.!)

and can calculate avg kinetic energy:

i.e.

$$\bar{E} \cong \int_0^{E_F} dE\, E\, n(E) \Big/ \int_0^{E_F} dE\, n(E)$$

$$\sim 1.1/r_e^2 \text{ (au)} \qquad \left(\sim \frac{\hbar^2}{2m} k_F^2\right)$$

in eV

→ note now have scales:

v_F, k_F, E_F of metal ($\sim 0°T$)

all in terms $\hbar$, m_e, n.

Now, what of better discussion of energy?

So far, have:

- Kinetic/repulsive energy

$$E \sim 1.1 / r_e^2 \qquad N r_e^3 \sim L^3$$

- Coulombic energy

idea (Drude-esque) is that ions ~ uniform, electrons ~ uniform so e.s. energy cancels,

⇒ this leaves K.E.

i-e attraction vs e-e repulsion ii } cancel.

- but, electrons undergo Pauli repulsion, i.e. so ↑↓ repulsion for opposite spins reduced.

⇒ Exchange Term

⇒ $\Delta E_{Pauli} = -\frac{.5}{r_e\,(au)}$ → $1/r_e$ ~~attractive~~ (force)

really decrease repulsion, opp. spins

⇒ thermal exchange → lowers energy.

$$E \simeq 1/r^2 - .5$$

→ Transport

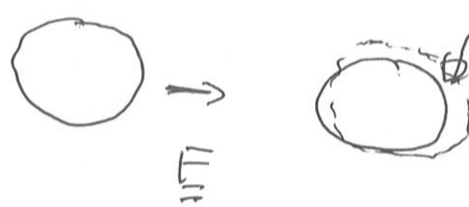
~ heuristics: For transport, need:
→ velocity
→ length, time

Here: $v \to v_F$

$\ell_{mfp} \to v_F \tau_{Relax}$.

Point: — only electrons on surface of Fermi sphere can move, in response $\underline{E}$. For $k < k_F$, all states occupied. Thus can't accelerate electrons, as states occupied. Opposing force is Pauli repulsion

— i.e.



$\underline{E}$

— 'surface layer' of electrons displaced by scattering!

— 'core' unchanged

To estimate:

$$\underline{p} = \hbar \underline{k}$$

$$\hbar \frac{d}{dt} \underline{k} = +e\underline{E}$$

$$\Delta \underline{k} \cong +e\underline{E}\,\Delta t \simeq +e\underline{E}\,\tau_{...}$$

Now $\underline{J} = Ne\, V_F \frac{\delta N}{N}$ — current carried by displaced surface Fermi electrons — $\delta N / N$
— speed is V_F, l.o.

$$\frac{\delta N}{N} \sim \frac{\Delta k}{k_F} \approx \frac{\Delta k_F}{k_F} \sim \frac{e\underline{E}\,\tau_{rlx}}{\hbar k_F}$$

$\hookrightarrow k_F^2 \Delta k / k_F^3$

$$V_F \cong \frac{\hbar k_F}{m_e}$$

$$\therefore \quad \underline{J} \sim \frac{Ne^2 \underline{E}\,\tau_{rlx}}{\hbar k_F} \frac{\hbar k_F}{m_e}$$

$$\boxed{\underline{J} \sim \frac{Ne^2 \tau_{rlx}}{m_e} \underline{E}}$$

$$\boxed{\sigma = \frac{Ne^2 \tau_{rlx}}{m_e}}$$

back to Drude, in form of answer!

But:

- current carried by Fermi sphere surface electrons, only.
- $V \rightarrow V_F$
- $\tau_{rlx} \rightarrow$ Fermi Golden Rule, QM transitions!

i.e.

$$1/\tau_{rlx} \sim \underset{\substack{\downarrow \\ \text{transition}}}{T_{fi}} \cong \frac{2\pi}{\hbar} \sum_{\underline{k}'} |\langle \underline{k} | \underset{\substack{\uparrow \\ \text{interaction}}}{V_{int}} | \underline{k}' \rangle|^2 \underset{\substack{\uparrow \\ \text{energy cons.}}}{\delta(\epsilon_{\underline{k}} - \epsilon_{\underline{k}'})} * (f_{\underline{k}'} - f_{\underline{k}})$$

Where:  $\rightarrow \dfrac{\underline{k}'}{\underline{k}}$ with $|\underline{k}| \sim |\underline{k}'|$

$\Rightarrow$ assures: $\epsilon_{\underline{k}} \sim \epsilon_{\underline{k}'}$

$f_{\underline{k}} \neq f_{\underline{k}'}$, as outside sphere.
$f \cong 1$ to $f = 0$ transition.

Similarly, for λ:

$$\lambda \sim n v\, l_{mfp} = \underbrace{C}_{\text{specific heat/vol.}} v_{th}\, l_{mfp} \rightarrow C\, V_F^2\, \tau_{rlx}$$

$\rightarrow$ does temp matter?! $\sim \dfrac{C}{m_e} E_F\, \tau_{rlx}$

Now, $\underbrace{C}_{\text{spec. heat}} = \partial \Delta E / \partial T$, defn.

— when thermal excitation applied, only a few electrons excited, those at Fermi energy by increment T.

so

$$-\ \Delta E \sim \overbrace{[n(E_F)\, T]}^{(N \Delta T)\ \#\text{ in layer}}\ \underbrace{T}_{\text{temp gain}}$$

$n(E)$ i.e.

$$\therefore \Rightarrow C \sim [n(E_F)\, T]$$

↳ specific heat is temp. dependent

$n(E_F)T$ — shell thickness

$$\Rightarrow \lambda \sim \frac{n(E_F)\, T\, E_F\, \tau_{rlx}}{m_e V}$$

So
$$\lambda \sim \frac{n(E_F)\, E_F T\, \tau_{rlx}}{m_e V}$$

For Weidemann-Franz:

$$\lambda/\sigma \sim \frac{n(E_F)\, E_F\, T\, \cancel{\tau_{rlx}}}{V\, \cancel{m_e}\, \frac{N e^2 \cancel{\tau_{rlx}}}{\cancel{m_e}}}$$

$$\sim \frac{n(E_F)\, E_F}{V N} \frac{T}{e^2} \qquad V \sim L^3$$

but

$$\frac{n(E_F)\, E_F}{L^3} \sim E_F \frac{\cancel{L^3}}{\cancel{L^3}} (E_F)^{1/2} \left(\frac{2m}{\hbar^2}\right)^{3/2}$$

need spec. heat/volume

$$\sim (E_F)^{3/2} \cancel{L^3} \left(2m/\hbar^2\right)^{3/2}$$

$$\sim \frac{N}{\cancel{L^3}} \cancel{L^3} \cancel{\left(2m/\hbar^2\right)^{3/2} \left(\frac{\hbar^2}{2m}\right)^{3/2}}$$

$$\sim N \to n \qquad c = C/V$$

$\therefore$
$$\boxed{\lambda/\sigma \sim \frac{\cancel{N} T}{\cancel{N} e^2}}$$

$\to$ $\boxed{\text{recover Weidemann-Franz!}}$

or
$$\lambda/\sigma \sim \frac{T}{e^2} k_B^2 \frac{\pi^2}{3}$$

$$\sim T \quad k_B^3 \pi^2 \to \text{Lorentz factor}$$

So — recover Weidemann-Franz!

Key: — τ_{relx} cancels

but — essential to note that transport dominated / due to electrons on surface of Fermi Sphere.

From Chapman-Enskog perspective:

recall for σ:

(MB)

$$-\frac{e}{m}\underline{E}\frac{\partial f}{\partial v} = C(f) = -\frac{1}{\tau_r}(f-f_0)$$

$$f = f_{0_{max}} + \delta f$$

$$\underline{J} = -n_0 e\int d^3v\, \underline{v}\, f$$

$$\therefore \quad -1/\tau_r(\delta f) \cong -\frac{e}{m_e}\underline{E}\cdot\frac{\partial f_0}{\partial \underline{v}}$$

(after $C-E$)

$$\delta f \cong \tau_r\frac{e}{m_e}\underline{E}\cdot\frac{\partial f_0}{\partial \underline{v}}$$

$\Rightarrow$

$$\underline{J} = -\frac{n_0 e^2}{}\int d^3v\, \tau_r\, \underline{E}\cdot\partial f_0\ v$$

so

$$\sigma \sim \frac{n_0 e^2}{m_0} \tilde{\tau}_{rlx}.$$

n.b. $\frac{\partial f_0}{\partial v} v \sim -\# f_0$

distributed over dof.

Now for QM particles, really need density matrix $\sum_{ij} |i\rangle\langle j|$. Can use $N(\underline{x}, \underline{k}, t)$

$\boxed{\text{Quasi-Particle Picture}}$

$N \Leftrightarrow$ Wigner Distribution Fctn.

i.e. $N = \int d^3\underline{p}\, e^{i\underline{p}\cdot\underline{x}}\, \psi^*(\underline{k}+\underline{p}/2)\, \psi(\underline{k}-\underline{p}/2)$ — wave fctns

classically $N \to$ action density

$\rightarrow$ population density

can view N as a kind of wave/wave function density

then

$$\frac{\partial N}{\partial t} + (\underline{v}_{gr} + \underline{V})\cdot\nabla N - \frac{\partial}{\partial \underline{x}}(\omega + \underline{k}\cdot\underline{V})\cdot\frac{\partial N}{\partial \underline{k}}$$

$$= -\frac{1}{\tau_{rlx}}(N - N_0)$$

Wave kinetic Equation

$$\frac{\partial N}{\partial t} + \frac{\partial \omega}{\partial k}\cdot\nabla N - \frac{\partial}{\cdot}\omega\cdot\frac{\partial N}{\cdot}$$

now $\quad \dfrac{\partial \omega}{\partial \underline{k}} \rightarrow \dfrac{\partial \hbar\omega}{\partial \hbar \underline{k}} \rightarrow \dfrac{\partial \epsilon}{\partial \underline{p}}$

$$-\frac{\partial \omega}{\partial \underline{x}} \cdot \frac{\partial}{\partial \underline{k}} \rightarrow -\frac{\partial \hbar\omega}{\partial x} \cdot \frac{\partial}{\partial \hbar \underline{k}} \rightarrow -\frac{\partial \epsilon}{\partial \underline{x}} \cdot \frac{\partial}{\partial \underline{p}}$$

but $\quad \epsilon = \epsilon_{kin} + V(\underline{x})$

$$-\frac{\partial \omega}{\partial \underline{x}} \cdot \frac{\partial}{\partial \underline{k}} \rightarrow -\frac{\partial V}{\partial \underline{x}} \cdot \frac{\partial}{\partial \underline{p}} \rightarrow \underline{F} \cdot \frac{\partial}{\partial \underline{p}}$$

$$\rightarrow \frac{\underline{F}}{\hbar} \cdot \frac{\partial}{\partial \underline{k}}$$

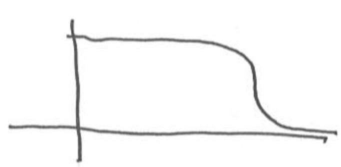
$$N \rightarrow g = g(\underline{x}, \underline{k}, t)$$

so

$$\boxed{\frac{\partial g}{\partial t} + \frac{\partial \epsilon}{\partial \underline{p}} \cdot \frac{\partial g}{\partial \underline{x}} + \frac{\underline{F}}{\hbar} \frac{\partial g}{\partial \underline{k}} = -\frac{1}{\tau_{rlx}} (g - g_0)}$$

For electrons in metals

$$g_0 = f_{FD}(E)$$



and

so

$$\cancel{\frac{\partial g}{\partial t}} + \cancel{\frac{\partial \epsilon}{\partial \underline{p}} \cdot \frac{\partial g}{\partial \underline{x}}} - \frac{e\underline{E}}{\hbar} \cdot \frac{\partial g}{\partial \underline{k}} = \frac{-1}{\tau_{rlx}} (g - g_0)$$

and now Chapman - Enskog, as usual.

$$g = g_0 + \delta g$$

$$-\frac{e\underline{E}}{\hbar} \cdot \frac{\partial g_0}{\partial \underline{k}} = \frac{-1}{\tau_{rlx}} \delta g$$

$$\underline{J} = -\frac{e}{m_e} \int d\underline{k}\, \hbar \underline{k}\, \delta g.$$

$g_0 \rightarrow$ even.

$$\delta g = + \tau_{rlx} \frac{e\underline{E}}{\hbar} \cdot \frac{\partial g_0}{\partial \underline{k}}$$

$$\boxed{\underline{J} = -\frac{e^2}{m_e} \int d\underline{k}\, \cancel{\hbar}\underline{k} \cdot \frac{1}{\cancel{\hbar}} \left(\frac{\partial g_0}{\partial \underline{k}} \cdot \underline{E} \right) \tau_{rlx}.}$$

but where is $\partial g_0 / \partial \underline{k} \neq 0$? $\rightarrow$ Fermi surface!

Now, $\underline{J} = -\frac{e^2}{m_e} \int d\underline{k}\, \tau_{rlx}\, \underline{k} \left[\frac{\partial g_0}{\partial \underline{k}} \cdot \underline{E} \right]$

$$\int d\underline{k} = \int 4\pi k^2 dk$$

$$\underline{k}\left[\underline{E}\cdot\frac{\partial g_0}{\partial \underline{k}}\right] \rightarrow \underline{k}(\underline{E}\cdot\underline{k})\frac{\hbar^2}{m}\frac{\partial g_0}{\partial E}$$

so

$$\underline{J} = -\frac{e^2}{m_e}\int 4\pi k^2 dk \; \tau_{rlx}\left[\underline{k}(\underline{E}\cdot\underline{k})\frac{\hbar^2}{m}\frac{\partial g_0}{\partial E}\right]$$

but $\frac{\hbar^2}{m} k\,dk = dE$

$$\boxed{\underline{J} = -\frac{e^2}{m_e}\int dE \; \tau_{rlx}\left[\underline{k}(\underline{E}\cdot\underline{k})\frac{\partial g_0}{\partial E}\right] k}$$

but $\partial g_0/\partial E \cong -\delta(E-E_F)$

$\therefore$

$$\boxed{\underline{J} \simeq \frac{e^2}{m_e}\tau_{rlx}\,\underline{\underline{E}}\; k_F^3}$$

$$\sigma \cong \frac{e^2}{\ }\tau_{relx}\, k_F^3$$

but $k_F^3 = 3\pi^2 \cancel{N} = \left(\left(N/L^3\right)^{1/3}\right)^3$

$$\boxed{\sigma \cong \frac{n e^2}{m_e} \tau_{rlx}}$$

as usual.

Messages:

- $\partial f_0 / \partial E = -\delta(E - E_F)$ captures idea of localization of electron response to Fermi surface.
- result same, physics different.
- can use WKE for wave transport problems much like Boltzmann Eqn.